

# Introduction to Fuzzy Logic

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# Overview

- Introduction
- Crisp Variables
- Fuzzy Variables
- Fuzzy Logic Operators
- Fuzzy Control
- Case Study

Fuzzy Logic

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# Introduction

- Fuzzy logic:
  - A way to represent variation or imprecision in logic
  - A way to make use of natural language in logic
  - Approximate reasoning
- Humans say things like "If it is sunny and warm today, I will drive fast"
- Linguistic variables:
  - Temp: {freezing, cool, warm, hot}
  - Cloud Cover: {overcast, partly cloudy, sunny}
  - Speed: {slow, fast}

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# Crisp (Traditional) Variables

- Crisp variables represent precise quantities:
  - $x = 3.1415296$
  - $A \in \{0,1\}$

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# Fuzzy Sets

A Fuzzy Set is a class with different degrees of membership. Almost all real world classes are fuzzy!  
Examples of fuzzy sets include: {'Tall people'}, {'Nice day'}, {'Round object'} ...

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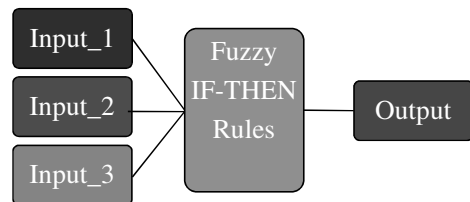
# Fuzzy Linguistic Variables

- Fuzzy Linguistic Variables are used to represent qualities spanning a particular spectrum
- Temp: {Freezing, Cool, Warm, Hot}
- Membership Function
- Question: What is the temperature?
- Answer: It is warm.
- Question: How warm is it?

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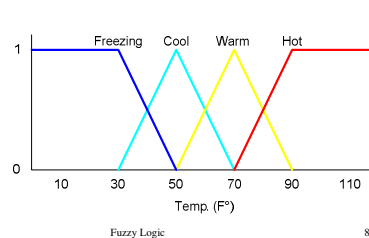
## Fuzzy Inference (Expert) Systems



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## Membership Functions

- Temp: {Freezing, Cool, Warm, Hot}
- Degree of Truth or "Membership"

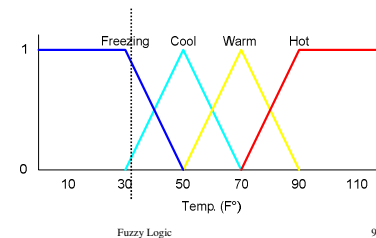


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## Membership Functions

- How cool is 36 F° ?

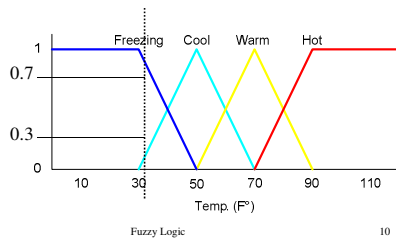


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## Membership Functions

- How cool is 36 F° ?
- It is 30% Cool and 70% Freezing



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## Fuzzy Logic

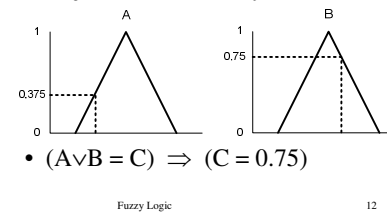
- How do we use fuzzy membership functions in predicate logic?
- Fuzzy logic Connectives:
  - Fuzzy Conjunction,  $\wedge$
  - Fuzzy Disjunction,  $\vee$
- Operate on degrees of membership in fuzzy sets

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## Fuzzy Disjunction

- $A \vee B \triangleq \max(A, B)$
- $A \vee B = C$  "Quality C is the disjunction of Quality A and B"

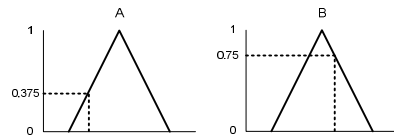


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## Fuzzy Conjunction

- $A \wedge B \triangleq \min(A, B)$
- $A \wedge B = C$  "Quality C is the conjunction of Quality A and B"



- $(A \wedge B = C) \Rightarrow (C = 0.375)$

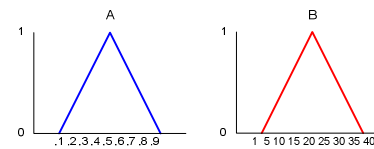
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## Example: Fuzzy Conjunction

Calculate  $A \wedge B$  given that A is .4 and B is 20



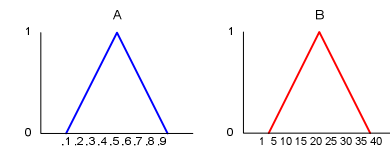
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## Example: Fuzzy Conjunction

Calculate  $A \wedge B$  given that A is .4 and B is 20



- Determine degrees of membership:

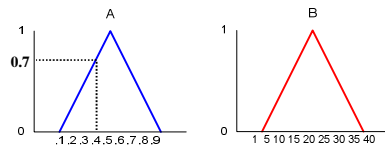
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## Example: Fuzzy Conjunction

Calculate  $A \wedge B$  given that A is .4 and B is 20



- Determine degrees of membership:
  - $A = 0.7$

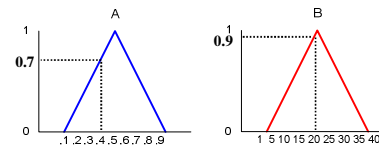
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## Example: Fuzzy Conjunction

Calculate  $A \wedge B$  given that A is .4 and B is 20



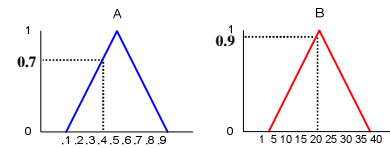
- Determine degrees of membership:
  - $A = 0.7$   $B = 0.9$

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## Example: Fuzzy Conjunction

Calculate  $A \wedge B$  given that A is .4 and B is 20



- Determine degrees of membership:
  - $A = 0.7$   $B = 0.9$
- Apply Fuzzy AND
  - $A \wedge B = \min(A, B) = 0.7$

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## Fuzzy Control

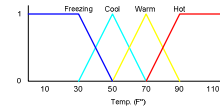
- Fuzzy Control combines the use of fuzzy linguistic variables with fuzzy logic
- Example: Speed Control
- How fast am I going to drive today?
- It depends on the weather.
- Disjunction of Conjunctions

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## Inputs: Temperature

- Temp: {Freezing, Cool, Warm, Hot}

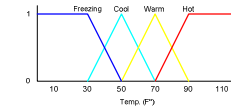


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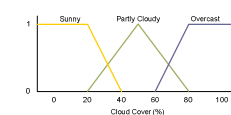
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## Inputs: Temperature, Cloud Cover

- Temp: {Freezing, Cool, Warm, Hot}



- Cover: {Sunny, Partly Cloudy, Overcast}

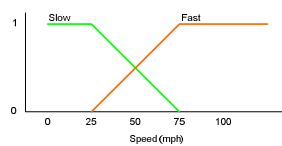


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## Output: Speed

- Speed: {Slow, Fast}



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## Rules

- If it's Sunny and Warm, drive Fast  
 $\text{Sunny}(\text{Cover}) \wedge \text{Warm}(\text{Temp}) \Rightarrow \text{Fast}(\text{Speed})$
- If it's Cloudy and Cool, drive Slow  
 $\text{Cloudy}(\text{Cover}) \wedge \text{Cool}(\text{Temp}) \Rightarrow \text{Slow}(\text{Speed})$
- Driving Speed is the combination of output of these rules...

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## Example Speed Calculation

- How fast will I go if it is
  - 65 F°
  - 25 % Cloud Cover ?

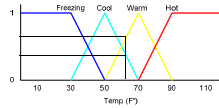
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### Fuzzification: Calculate Input Membership Levels

- 65 F° ⇒ Cool = 0.4, Warm = 0.7

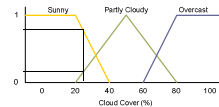
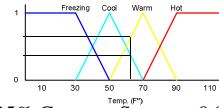


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### Fuzzification: Calculate Input Membership Levels

- 65 F° ⇒ Cool = 0.4, Warm = 0.7
- 25% Cover ⇒ Sunny = 0.8, Cloudy = 0.2



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### ...Calculating...

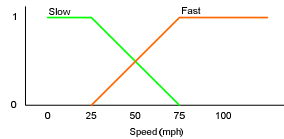
- If it's Sunny and Warm, drive Fast  
 $Sunny(Cover) \wedge Warm(Temp) \Rightarrow Fast(Speed)$   
 $0.8 \wedge 0.7 = 0.7$   
 $\Rightarrow \mathbf{Fast = 0.7}$
- If it's Cloudy and Cool, drive Slow  
 $Cloudy(Cover) \wedge Cool(Temp) \Rightarrow Slow(Speed)$   
 $0.2 \wedge 0.4 = 0.2$   
 $\Rightarrow \mathbf{Slow = 0.2}$

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### Defuzzification: Constructing the Output

- Speed is 20% Slow and 70% Fast



- Find centroids: Location where membership is 100%

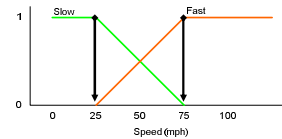
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### Defuzzification: Constructing the Output

- Speed is 20% Slow and 70% Fast



- Find centroids: Location where membership is 100%

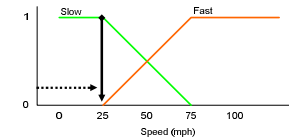
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### Defuzzification: Constructing the Output

- Speed is 20% Slow and 70% Fast



- Speed = weighted mean  
 $= (2 * 25 + \dots)$

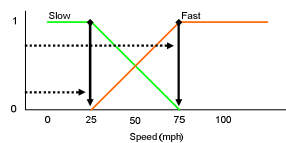
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## Defuzzification: Constructing the Output

- Speed is 20% Slow and 70% Fast



- Speed = weighted mean  
=  $(2 * 25 + 7 * 75) / (9)$   
= 63.8 mph

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## Notes: Follow-up Points

- Fuzzy Logic Control allows for the smooth interpolation between variable centroids with relatively few rules
- This does not work with crisp (traditional Boolean) logic
- Provides a natural way to model some types of human expertise in a computer program

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## Notes: Drawbacks to Fuzzy logic

- Requires tuning of membership functions
- Fuzzy Logic control may not scale well to large or complex problems
- Deals with imprecision, and vagueness, but not uncertainty

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## Summary

- Fuzzy Logic provides way to calculate with imprecision and vagueness
- Fuzzy Logic can be used to represent some kinds of human expertise
- Fuzzy Membership Sets
- Fuzzy Linguistic Variables
- Fuzzy AND and OR
- Fuzzy Control

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